Home assignment\_4\_ Reference

1. Observability
   1. Check the observability:

See the observability matrix , using matlab command

clear all;clc;

A =[1 1;4 2];

C = [1 0];

Q =obsv(A,C)

rank(Q)

* Since , if , is an observable.
* The rank should be less than ..
  1. Symbolic math:

Now the system is

Then for any check the observability

clear all;clc;

syms a b

A =[0 1; a b];

C = [1 1];

Q = [C; C\*A]

rank(Q)

* 1. A familiar form of a system as

- Check the observability

- how about observability if

1. Optimal observer design in a scalar system – Example 4.3 in textbook
   1. Generate the noises with intensity

clear all; clc;clf

V1= 0.1;V2 = 0.01; % noise intensity

x0 = 1;

v1 = V1\*randn(1000,1) ; % or v1 = V1\*wgn(1000,1,0);

v2 = V2\*randn(1000,1); % or v1 = V2\*wgn(1000,1,0);

t = linspace(0,5,1000);

figure(1)

subplot(1,2,1)

plot(t,v1,'r', t,v2,'b'); grid on;

title('disturbace w1 and noise w2')

* 1. Generate a simulated system
     1. find the sate space model in matlab

% system dynamics

A= -1; B = 1 ; C=1;D = 0;

sysx = ss(A,B,C,D);

[y,t,x]=lsim(sysx, v1, t,1);

yout = y + v2;

subplot(1,2,2)

plot(t,yout,t,y); grid on

title('the original state with disturbance and output with noise')

%% check matlab commands : ss, lsim

* 1. The optimal observer design
     1. Solving Riccati equation

% solving Riccati equation

syms Q

eqn = 2\*A\*Q + V1 - Q^2/V2;

Ri = double(solve(eqn,Q)) % for Q>= 0, Ri(2) is validated.

* + 1. The optimal observer gain

% the optimal observer gain

ObK = Ri(2)/V2

AO= A-ObK;

BO=[1 ObK];

uO = [v1 y];

eig(AO)

sysO = ss(AO,BO,C,[ ]);

Temp=lsim(sysO,uO,t,0);

figure(3)

plot(t,yout,'b', t,Temp,'r',t,y,'k'); grid on

1. Optimal observer design in a scalar system – Example 4.4 (continue example 2.4)in textbook
   1. Problem

* DC motor

The is assumed a **white noise** with intensity

%%%%% ---------white noise and colored noise

Given a physical system, to model it to a mathematical description, there are known and unknown parameters. In general unknown parameters are modeled disturbance or noise.

The difference between disturbance and noise is the characteristic of the fluctuation in time.

First in the case noise, let’s us to measure a voltage of a resistor using a oscilloscope. Then the measured voltage is not only constant but varies rapidly even if its alternating value is small. This is due to many facts such as thermal effects, electromagnetic interferences and so on. We may model these unknown as a white noise. This white noise is all around in measurement system as well as in the system dynamic unknown parameters.

Second in the case of disturbance. Let’s see an inverted pendulum problem. It is stayed for a moment.

Now you push or touch the pendulum with a some force, which is input to the pendulum, it is unknown the magnitude of your force and the time when you push the pendulum. In this case your force which is may be modeled as a disturbance. Another example, Let’s see a rotating motor. When it is rotating, an unknown load or torque is applied your motor, we may model as a disturbance .You may notice the difference between noise and disturbance. In mathematically, the white noise is of one of the famous properties is

If you transfer using fourier transform, in the frequency domain the white noise is in all the frequency range!! So we call it as “White “. Why do they call it a white? Since the light is white even if it is summed all of the different frequencies!!

Now the disturbance has a property that its frequency range is limited. If you push the pendulum, your force is frequency limited. Sure? Yap because you can not push the pendulum with 1000Hz !! So the engineer modeled your push force is a frequency band limited noise. Let’s transform the disturbance using fourier transform as

The engineer is called this disturbance as a colored noise. Why do they call it as a “colored noise”? You may check the frequency of “a yellow light” which is higher than that of a ‘red light’. So the band limited noise as a ‘colored noise’, and the disturbance is modeled as a colored noise. In fact in the above example, is modeled as a colored noise in general. If you want to know noise in detail, you may google it or talk to me.

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* 1. The state equation

The state space model with simplifying parameters

Where

and the process noise, in general it is a colored noise, but this time it is assumed a white noise

with intensity (unbiased as in matlab)

And measurement noise is a white noise(unbiased as in matlab)

When a physical system is to be a mathematical model, there is a tedious procedure but **IT is**

**Important. The value parameters should be consistent.** If the parameter value in mks, then

every parameter should be in mks. It should be. For example let’s see

the unit is [kg]^-1 [m]^-2. Since kg = 1000g, you may not use as

which is **a totally different value**. And in the real situation, the data given to us are not consistent.

Hence you should be careful to handle it.

Since the state space model is not a standard form, I will change it as

Or you may keep the original one.

% system dynamics

a= 4.6;

gam = 0.1;

Vdg =gam^2\*10; % the intensity of the disturbance

Vm =10^-7; % the noise intensity

%ka = 0.787;

% the plant state space model

A = [ 0 1;0 -a];

B = [0 1]';

C = [1 0];

sys= ss(A,B,C,0);

Since the plant is marginally stable(?), the output is not converge to zero. I want to compare

with / without disturbance.

% an trajectory example with an initial point

x0 =[1;1];

y0 = C\*x0; % inital values of the plant states

N = 1000; % the sampling number of data

tf = 10 ; % the final time

t = linspace(0,tf,N);

vd = Vdg\*randn(N,1); % gaussian noise ... if awgn is available, use agwn (it stands for

% additive white gaussian noise

[y,t,x]=lsim(sys,zeros(N,1), t,x0); % without disturnbance

[yd,t,x]=lsim(sys,vd, t,x0); % disturnbance

figure(1)

plot(t,y,'r', t,yd,'b'); grid on

title('without disturbace and with disturbance')

axis([0 tf 0.8 2])

% the plant output corrupted by a noise

vm = Vm\*randn(N,1);

ym = yd + vm;

figure(2)

plot(t,yd,'b', t,ym, 'r'); grid on

axis([0 tf 0.8 2])

title('without(blue) and with (red) measurement noise')

In figure(1), with the initial point(1,1), you may see the trajectory without and with disturbance. As you see, the plant is not asymptotic stable, hence trajectories do not converge to zeros.

In figure(2), without/with measurement , the trajectories are not too different. Yap. Since the intensity of noise is is too small. If you change the intensity of the measurement noise, you may see the effect due to the measurement noise.

* 1. Matlab command “lqe”

There are two different matlab commands to help to design the optimal obserever.

One is ‘lqe’, stands for ‘linear quadratic estimator’, and the others is ‘care’ for ‘ continuous algebraic riccati equation’. In order use one of these commands, you should follow the matlab descriptions.

Let’s see ‘lqe’ command in matlab

>> help lqe

lqe Kalman estimator design for continuous-time systems.

Given the system

.

x = Ax + Bu + Gw {State equation}

y = Cx + Du + v {Measurements}

with unbiased process noise w and measurement noise v with

covariances

E{ww'} = Q, E{vv'} = R, E{wv'} = N ,

[L,P,E] = lqe(A,G,C,Q,R,N) returns the observer gain matrix L

such that the stationary Kalman filter

.

x\_e = Ax\_e + Bu + L(y - Cx\_e - Du)

produces an optimal state estimate x\_e of x using the sensor

measurements y. The resulting Kalman estimator can be formed

with ESTIM.

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and the estimator poles E = EIG(A-L\*C).

1. OK.. what is an unbiased process noise w and measurement noise? An unbiased noise in matlab is different form the mathematic definition.

In math, an unbiased noise /estimator /signal is defined as

In matlab it is defined as

) = 0

1. In what are these parameters ? They are already defined in matlab so that you should put these parameters in order as in maltab.

Another matlab command for the optimal observer may be ‘care’, which refers to a

‘continuous algebraic riccati equation’. But if you use this command you should be careful

to substitute parameters in the matlab format. I will use ‘lqe’. I think old version of matlab

did not provide ‘lqe’ command but ‘care’ command.

* 1. Check the observability..Please check
  2. Find the optimal observer gain

%% design the optimal observer gain

[L,P,E] = lqe(A,B,C,Vdg,Vm)

OK.. It’s nice? What is ‘L,P,E’ ? check to help lqe. The gain matrix is “L” as

L =

40.3573

814.3564

Hence you get the optimal gain for observer. Is it finished? No you should confirm your results by computer simulation.

* 1. Computer simulation

To simulate your results, there are two ‘BIG’ models.

* The real – plant – model
* The observer model in realized in computer.

Hmmm…If the real plant has a state whose number is , then the observer state is of the number is . So in simulation, there are states(in previous scalar example, I deliberately manage it.)

First the plant is

Second the observer is

There are two state vectors .Let’s define the new sate as

Then

Hmm. This is the total system with the optimal observer. Remember in the real plant, the measured state is , of which sate number is less than that of the plant state, but the number of the observer output is the same as that of the plant state. This means you may observe /estimate all the plant state using the plant output.

The total simulation model is

% Total system

AT = [A zeros(2,2) ; L\*C A-L\*C];

BT = [ B B zeros(2,2); zeros(2,2) B L];

CT = [C zeros(1,2); zeros(2,2) eye(2,2)];

Now we may “observer” or “estimate” the full state using the output. Since we do not know the initial points, the initial points of the observer states are assumed to be .

Ok. In figure(3), there are three trajectories. The red one is “the observed the state using Kalman estimator”. Later with these observers, we may design the full state feedback gain.

This is the essential to “Linear optimal controller”.

* 1. Some comments.

To design the observer, we should know the plant model parameters, which is the same to the observer system. Hence these parameters of the observer is not the same to the plant, then the observer is not the optimal. How the engineer try to solve it?...Hmm this is the another subject as the uncertainty problem. There are huge materials on this subject.

And if you estimate only some states of the plants, how to design the optimal observer in this case?...Hmm I think this problem is interesting, and there should be some approach. Check it.

The estimator is good subjects.

You may use it not only control problem, but in data science as face recognition, classification,… and you may predict unknowns which is the most important in discrete system. How can you predict the trend of this deadly corona virus? Hope some of you can do that in the future.

1. Home assignment\_4

Given systems and parameters

,

* 1. Design the optimal observer gain .
  2. Assume the control

Find the observer trajectories with

* 1. Assume the control

Find the observer trajectories with

Good luck.